CNC Router Calculations

Initial Data

Machine Rapid Speed (v) 20 m/min	**Acceleration Time (t)** 0.5 sec
X Axis Mass 48.25 kg	**Y Axis Mass** 15.66 kg
Z Axis Mass 8.56 kg	**X/Y Axis Pinion Pitch Radius (P_r)** 15 mm
Gearing Ratio (G_r) 5:1	**Gearing Efficiency (G_e)** 90%
Z Axis Ball Screw Lead (P) 10 mm	**Ball Screw Efficiency (e)** 90%
Linear Guide Intrinsic Resistance (F_g) 10 N per guide	**Cutting Force (assumed) (F_c)** 200 N

Assumptions/Details

- Each linear guide exhibits 10 N of intrinsic resistance which, for simplicity, includes any static/dynamic friction. The X axis has four linear guides, while the Y and Z axes have two guides each.
- A cutting force of 200 N has been included as an assumed maximum force on each axis in all directions of motion.
- The X axis is driven by two motors, while the Y and Z axes are driven by one motor each.

Acceleration Calculations

Acceleration Rate (a_c) $a_c = \frac{v}{t}$ $= \frac{\frac{20}{60}}{0.5}$ $= 0.667\ m/s^2$	**Acceleration Displacement (s)** $s = s_0 + v_o t + \frac{1}{2} a_c t^2$ $= 0 + 0 + \frac{1}{2} \cdot 0.667 \cdot 0.5^2$ $= 0.083\ m$

Linear Force Calculations (Using the Principle of Work and Energy)

Principle of Work and Energy $T_1 + \sum U_{1-2} = T_2$ Where $T = \frac{1}{2} m v^2$	**Work of Weight** $U_{1-2} = W \Delta y$ Where Δy is the change in height from the initial to final position of the component in motion. Here, $\Delta y = s = 0.083\ m$.
Work of a Constant Force $U_{1-2} = F \cos \emptyset (s_2 - s_1)$ Where s_1 is the initial and s_2 is the final position of the component in motion. Here, $s_2 - s_1 = 0.083\ m$. Due to collinear motion, $\cos \emptyset = \cos 0 = 1$	

Z Axis Rapid "Upwards" Movement (F_{zr})

$$T_1 - W\Delta y - F_g \cos\emptyset(s_2 - s_1) + F_{zr} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 8.56 \cdot 0^2 - 8.56 \cdot 9.81 \cdot 0.083 - 2 \cdot 10 \cdot 0.083 + F_{zr} \cdot 0.083 = \frac{1}{2} \cdot 8.56 \cdot \frac{20^2}{60}$$

$$\therefore F_{zr} = 109.7\ N$$

Z Axis Rapid "Downwards" Movement with Cutting Force (F_{zc})

$$T_1 - W\Delta y + F_g \cos\emptyset(s_2 - s_1) + F_c \cos\emptyset(s_2 - s_1) - F_{zc} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 8.56 \cdot 0^2 - 8.56 \cdot 9.81 \cdot 0.083 + 2 \cdot 10 \cdot 0.083 + 200 \cdot 0.083 - F_{zc} \cdot 0.083 = \frac{1}{2} \cdot 8.56 \cdot \frac{20^2}{60}$$

$$\therefore F_{zc} = 130.3\ N$$

Y Axis Rapid Movement (F_{yr})

$$T_1 - F_g \cos\emptyset(s_2 - s_1) + F_{yr} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 15.66 \cdot 0^2 - 2 \cdot 10 \cdot 0.083 + F_{yr} \cdot 0.083 = \frac{1}{2} \cdot 15.66 \cdot \frac{20^2}{60}$$

$$\therefore F_{yr} = 30.5\ N$$

Y Axis Rapid Movement with Cutting Force (F_{yc})

$$T_1 - F_g \cos\emptyset(s_2 - s_1) - F_c \cos\emptyset(s_2 - s_1) + F_{yc} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 15.66 \cdot 0^2 - 2 \cdot 10 \cdot 0.083 - 200 \cdot 0.083 + F_{yc} \cdot 0.083 = \frac{1}{2} \cdot 15.66 \cdot \frac{20^2}{60}$$

$$\therefore F_{yc} = 230.5\ N$$

X Axis Rapid Movement (F_{xr})

$$T_1 - F_g \cos\emptyset(s_2 - s_1) + F_{xr} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 48.25 \cdot 0^2 - 4 \cdot 10 \cdot 0.083 + F_{xr} \cdot 0.083 = \frac{1}{2} \cdot 48.25 \cdot \frac{20^2}{60}$$

$$\therefore F_{xr} = 72.3\ N$$

X Axis Rapid Movement with Cutting Force (F_{xc})

$$T_1 - F_g \cos\emptyset(s_2 - s_1) - F_c \cos\emptyset(s_2 - s_1) + F_{xc} \cos\emptyset(s_2 - s_1) = T_2$$

$$\frac{1}{2} \cdot 48.25 \cdot 0^2 - 4 \cdot 10 \cdot 0.083 - 200 \cdot 0.083 + F_{xc} \cdot 0.083 = \frac{1}{2} \cdot 48.25 \cdot \frac{20^2}{60}$$

$$\therefore F_{xc} = 272.3\ N$$

*Table of Linear Forces**

Z Axis Rapid "Upwards" Movement (F_{zr})	109.7 N
Z Axis Rapid "Downwards" Movement with Cutting Force (F_{zc})	130.3 N
Y Axis Rapid Movement (F_{yr})	30.5 N
Y Axis Rapid Movement with Cutting Force (F_{yc})	230.5 N
X Axis Rapid Movement (F_{xr})	72.3 N
X Axis Rapid Movement with Cutting Force (F_{xc})	273.3 N

*Maximum linear force values are used to calculate axis peak motor torque requirements.

Z Axis Peak Motor Torque Requirements (Z_t)
Ball Screw Formula:

$$Z_t = \frac{F_{zc} \cdot P}{2 \cdot \pi \cdot e}$$

$$= \frac{130.3 \cdot 0.010}{2 \cdot \pi \cdot 0.9}$$

$$= 0.230\ N.m$$

Including 100% Factor of Safety:

$$\boldsymbol{Z_t = 0.460\ N.m}$$

Y Axis Peak Motor Torque Requirements (Y_t)
Rack and Pinion Formula:

$$Y_t = \frac{F_{yc} \cdot P_r}{G_r \cdot G_e}$$

$$= \frac{230.5 \cdot 0.015}{5 \cdot 0.9}$$

$$= 0.768\ N.m$$

Including 100% Factor of Safety:

$$\boldsymbol{Y_t = 1.537\ N.m}$$

X Axis Peak Motor Torque Requirements (X_t)
Rack and pinion formula:

$$X_t = \frac{F_{xc} \cdot P_r}{G_r \cdot G_e}$$

$$= \frac{273.3 \cdot 0.015}{5 \cdot 0.9}$$

$$= 0.911\ N.m$$

Dividing result across two X axis motors:

$$X_t = 0.455\ N.m$$

Including 100% Factor of Safety:

$$\boldsymbol{X_t = 0.911\ N.m}$$

Table of Peak Motor Torque Requirements

X Axis	0.911 N.m
Y Axis	1.537 N.m
Z Axis	0.460 N.m