This should give some insight on how the kinematics for ‘xyzab-drt.comp’ where derived following the separate document “5-Axis Machine Tools: Kinematics and Vismach Implementation in LinuxCNC” by Rudy du Preez.

Forward transformation

The transformation can be defined by the sequential multiplication of the matrices:

$$^{w}A_{t}={}^{w}A_{B}\cdot{}^{B}A_{A}\cdot{}^{A}A_{P}\cdot{}^{P}A_{t} \tag{18}$$

with the matrices built up as follows:

$$^{W}A_{B}=\begin{bmatrix}1&0&0&L_x\\0&1&0&L_y\\0&0&1&L_Z\\0&0&0&1\end{bmatrix} \qquad {}^{B}A_{A}=\begin{bmatrix}C_B&0&S_B&0\\0&1&0&0\\-S_B&0&C_B&0\\0&0&0&1\end{bmatrix} \tag{19}$$

$$^{A}A_{P}=\begin{bmatrix}1&0&0&0\\0&C_A&-S_A&0\\0&S_A&C_A&0\\0&0&0&1\end{bmatrix} \qquad {}^{P}A_{T}=\begin{bmatrix}1&0&0&P_x\\0&1&0&P_y\\0&0&1&P_z\\0&0&0&1\end{bmatrix} \tag{20}$$

Executing the matrix multiplications starting from the left:

$$^{w}A_{A}={}^{w}A_{B}\cdot{}^{B}A_{A}$$

$$^{w}A_{A}=\begin{bmatrix}1&0&0&L_x\\0&1&0&L_y\\0&0&1&L_z\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}C_B&0&S_B&0\\0&1&0&0\\-S_B&0&C_B&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}C_B&0&S_B&L_x\\0&1&0&L_y\\-S_B&0&C_B&L_z\\0&0&0&1\end{bmatrix}$$

$$^{w}A_{P}={}^{w}A_{A}\cdot{}^{A}A_{P}$$

$$^{w}A_{P}=\begin{bmatrix}C_B&0&S_B&L_x\\0&1&0&L_y\\-S_B&0&C_B&L_z\\0&0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0\\0&C_A&-S_A&0\\0&S_A&C_A&0\\0&0&0&1\end{bmatrix}=\begin{bmatrix}C_B&S_AS_B&S_BC_A&L_x\\0&C_A&-S_A&L_y\\-S_B&S_AC_B&C_AC_B&L_z\\0&0&0&1\end{bmatrix}$$

$$^{w}A_t = {}^{w}A_P \cdot {}^{P}A_t$$

$$^{w}A_t = \begin{bmatrix} C_B & S_A S_B & S_B C_A & L_x \\ 0 & C_A & -S_A & L_y \\ -S_B & S_A C_B & C_A C_B & L_z \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{w}A_t = \begin{bmatrix} C_B & S_A S_B & S_B C_A & C_B P_x + S_A S_B P_y + S_B C_A P_z + L_x \\ 0 & C_A & -S_A & C_A P_y - S_A P_z + L_y \\ -S_B & S_A C_B & C_A C_B & -S_B P_x + S_A C_B P_y + C_A C_B P_z + L_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

$$Q = \begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{bmatrix} = \begin{bmatrix} C_B P_x + S_A S_B P_y + S_B C_A P_z + L_x \\ C_A P_y - S_A P_z + L_y \\ -S_B P_x + S_A C_B P_y + C_A C_B P_z + L_z \\ 1 \end{bmatrix} \quad (25)$$

This represents the Forward Kinematic : $\begin{array}{l} Q_x = C_B P_x + S_A S_B P_y + C_A S_B P_z + L_x \\ Q_y = C_A P_y - S_A P_z + L_y \\ Q_z = -S_B P_x + S_A C_B P_y + C_A C_B P_z + L_z \end{array}$

$$Q = \begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{bmatrix} = \begin{bmatrix} C_B & S_A S_B & S_B C_A & L_x \\ 0 & C_A & -S_A & L_y \\ -S_B & S_A C_B & C_A C_B & L_z \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = {}^{Q}A_P \cdot P \quad (26)$$

Inverse transformation

Using $$^{q}A_{p}=\begin{bmatrix} R & q \\ 0 & 1 \end{bmatrix}$$

and (26)

We extract the rotation matrix *R* and the translation vector *q:*

$$R=\begin{bmatrix} C_B & S_A S_B & S_B C_A \\ 0 & C_A & -S_A \\ -S_B & S_A C_B & C_A C_B \end{bmatrix} \qquad q=\begin{bmatrix} L_x \\ L_y \\ L_Z \end{bmatrix}$$

Using $$\left({}^{q}A_{p}\right)^{-1}=\begin{bmatrix} R^T & -R^T q \\ 0 & 1 \end{bmatrix}$$

$$-R^T=\begin{bmatrix} -C_B & 0 & S_B \\ -S_A S_B & -C_A & -S_A C_B \\ -S_B C_A & S_A & -C_A C_B \end{bmatrix}$$

$$-R^T q=\begin{bmatrix} -C_B & 0 & S_B \\ -S_A S_B & -C_A & -S_A C_B \\ -S_B C_A & S_A & -C_A C_B \end{bmatrix}\cdot\begin{bmatrix} L_x \\ L_y \\ L_Z \end{bmatrix}=\begin{bmatrix} 0 \\ -S_A S_B L_x \;-\; C_A L_y \;-\; S_A C_B L_z \\ -S_B C_A L_X \;+\; S_A L_y \;-\; C_A C_B L_z \end{bmatrix}$$

$$P=\left({}^{Q}A_{P}\right)^{-1}\cdot Q=\begin{bmatrix} R^T & -R^T q \\ 0 & 1 \end{bmatrix}\cdot Q$$

$$P=\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}=\begin{bmatrix} C_B & 0 & -S_B & 0 \\ S_A S_B & C_A & S_A C_B & -S_A S_B L_x \;-\; C_A L_y \;-\; S_A C_B L_z \\ S_B C_A & -S_A & C_A C_B & -S_B C_A L_X \;+\; S_A L_y \;-\; C_A C_B L_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{bmatrix} \quad (27)$$

$$\begin{aligned} P_x &= C_B\left(Q_x-L_x\right) \;-\; S_B\left(Q_z-L_z\right) \\ P_y &= S_A S_B\left(Q_x-L_x\right) \;+\; C_A\left(Q_y-L_y\right) \;+\; S_A C_B\left(Q_z-L_z\right) \\ P_z &= S_B C_A\left(Q_x-L_x\right) \;-\; S_A\left(Q_y-L_y\right) \;+\; C_A C_B\left(Q_z-L_z\right) \end{aligned} \quad (28)$$